

Midterm 1

1. (21 points) For all the parts to this problem, let the annual discount rate be 5%.

a) Find the present value of the following cashflow: receive \$10 every year for 30 years with the first payment being 10 years from now.

$$\text{Answer: } 10/.05*(1-(1.05)^{-30})/1.05^9 = \$99.09$$

b) Find the present value of the following cashflow: receive \$10m now and the same amount a year from today and pay \$3m a year forever with the first payment being a year from today.

$$\text{Answer: } 10m+10m/1.05-3m/.05 = -\$40.48m$$

c) Consider the following two cashflows. For cashflow A, you receive \$10 every year for 5 years with the first payment being today. For cashflow B, you receive x dollars every year forever with the first payment being today. What is the value of x in order for cashflow B to have the same present value as cashflow A?

$$\text{Answer: } 10+10/.05*(1-1.05^{-4})=x+x/.05 \Rightarrow x = \$2.16$$

2. (15 points) Today, you're in charge of the nation's finances. Suppose that projected 2015 shortfall is \$418 billion and projected 2030 shortfall is \$1,345 billion. In present value terms, how large is the difference of the two budget shortfalls? Assume a 3% discount rate.

$$\text{Answer: } |418b/1.03^4-1345b/1.03^{19}| = \$396 \text{ billion}$$

3. (18 points) Suppose that you borrowed \$20k for 36 months to buy a car last year at an annual interest rate of 5% compounded monthly.

a) What is the amount of monthly payment?

$$\text{Answer: } 20k*0.05/12/(1-(1+0.05/12)^{-36}) = \$599.42$$

b) Calculate the effective annual interest rate for both the car loan and for a rate of 6% compounded quarterly. Which is larger?

$$\text{Answer: for car loan: } (1+0.05/12)^{12} - 1 = 5.12\%$$

$$\text{For 6\% compounded quarterly: } (1+0.06/4)^4 - 1 = 6.14\%$$

c) You made monthly payments for the last 12 months. But you still have to make 24 more payments. What is the present value of the remaining payments?

$$\text{Answer: } 599.42/(.05/12)*(1-(1+0.05/12)^{-24})/(1+0.05/12)^{12} = \$12,998$$

(or \$13,663 for PV at year 12)

4. (15 points) Suppose that you consider some mortgage options. The price of home is \$200k. Calculate your monthly payments for each option:
- Option A: 20% down payment at 15-year fixed annual rate of 4%
 - Option B: 15% down payment at 30-year fixed annual rate of 4.5%
 - Option C: 10% down payment at 30-year fixed annual rate of 6%

Answer: Option A: $160000 * .04/12 / (1 - (1 + .04/12)^{-180}) = \1183.50

Option B: $170000 * .045/12 / (1 - (1 + .045/12)^{-360}) = \861.37

Option C: $180000 * .06/12 / (1 - (1 + 0.06/12)^{-360}) = \1079

5. (15 points) Suppose that an account has \$6m now. The money is invested and obtains a return of 2%. Your business projections are that in year one you take out \$2m, in year two you take out \$0.7m, in year three you add \$1m to the account, and in year four you add \$4m to the account. Calculate the amount of money in the account a year from now, two years from now, three years from now, and four years from now.

Answer:

Just after the interest payment and before the external transaction

Year 1 \$6.12m

Year 2 \$4.2024m

Year 3 \$3.5724m

Year 4 \$4.6639m

Just after the external transaction

Year 1 \$4.12m

Year 2 \$3.5024m

Year 3 \$4.5724m

Year 4 \$8.6639m

6. (16 points) Consider a 30-year mortgage with a 5% interest rate and a 20% down payment. If you can afford a \$1000 monthly payment, how expensive a house can you buy?

Answer: $1000 = x * 0.05/12 / (1 - (1 + .05/12)^{-360}) \Rightarrow x = 186280$

You can afford at most $x/0.8 = \$232,850$